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RESONANT INTERACTIONS OF SURFACE AND INTERNAL GRAVITY WAVES.(U)
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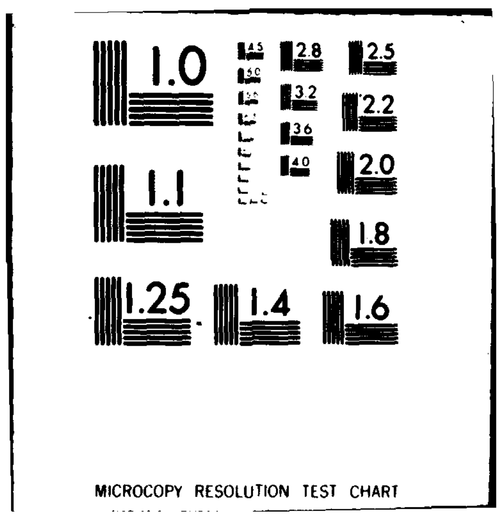
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(6) Resonant Interactions of

Surface and Internal Gravity Waves.

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A stably stratified, two-layer fluid system with a free surface admits resonant triads, including ones involving one surface and two internal waves.

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Resonant triads that couple surface and internal gravity waves are thought to play an important role in transferring energy from wind-generated surface waves to internal waves. Ball[1] used a geometric construction to show that the linearized dispersion relation for a two-layer liquid system with a free upper surface and no stream velocity ~~(see Figure 1)~~ admits resonant triads. On the basis of the required kinematic conditions,

$$\vec{k}_1 + \vec{k}_2 = \vec{k}_3, \qquad \omega_1 + \omega_2 = \omega_3, \tag{1}$$

where $\vec{k}_j$ denotes the horizontal wavenumber and ω_j the frequency of the j-th wave, Ball[1] claimed that the triads in question "always involve two external [i.e., surface] waves ... and one internal wave". This claim was supported by Phillips[2]: "...the interaction of two internal waves to generate a surface wave, does not seem to be possible". Again, his argument was based on the impossibility of satisfying the kinematic conditions, (1). The purpose of this note is to demonstrate that the two-layer model admits resonant triads involving two internal waves and one surface wave.

The linearized dispersion relation for the two fluid model is[3]:

$$\begin{aligned}\omega^4[1 + (1-\Delta)\tanh kH \tanh kh] - \omega^2 gk[\tanh kH + \tanh kh] \\ + \Delta g^2k^2 \tanh kH \tanh kh = 0,\end{aligned} \tag{2}$$

(notation in Fig. 1). If this configuration is to represent the open ocean, we may let $H \to \infty$ without practical loss. In this limit, the dispersion relation may be factored in simple terms:

$$(\omega^2 - g|k|)\left(\omega^2 - \frac{\Delta gk \tanh kh}{1 + (1-\Delta) \tanh |k|h}\right) = 0 . \qquad (3)$$

Clearly the first factor represents the surface waves, while the second represents the internal waves. The solution-curves of (3) are sketched in Fig. 2.

Next we use Ball's geometric method to demonstrate the existence of the triad in question. Choose any point (A) on the curve OI_1 in Fig. 2. A curve, commencing at A, drawn congruent to OI_2, intersects OS_1 at B. The vector commencing at B, parallel to $\overrightarrow{AO}$, intersects OI_2 at C. The points A, B, C form a resonant triad of solutions of (3). This triad contains two internal waves, A and C, and one surface wave, B.

To make the point even more strongly, we may let $h \to \infty$ in (3), which then reduces to

$$(\omega^2 - g|k|)\left(\omega^2 - \frac{\Delta g|k|}{2-\Delta}\right) = 0 . \qquad (4)$$

In this case, the triad in question may be given in closed form. For any real Ω,

$$\begin{aligned}
\omega_1 &= \frac{1}{1-\Delta}\,\Omega , & g k_1 &= \frac{(2-\Delta)}{\Delta(1-\Delta)^2}\,\Omega^2 , \\
\omega_2 &= \Omega , & g k_2 &= -\frac{(2-\Delta)}{\Delta}\,\Omega^2 , \qquad (5)\\
\omega_3 &= \frac{2-\Delta}{1-\Delta}\,\Omega , & g k_3 &= \frac{(2-\Delta)^2}{(1-\Delta)^2}\,\Omega^2 .
\end{aligned}$$

It is apparent that this triad satisfies both (1) and (4), and that k_1 and k_2 represent internal waves, while k_3 represents a surface wave.

The assumption that $H \to \infty$ in (2) is not essential. The construction in Fig. 2 applies equally well to the solution curves of (2). The restriction to a two-layer system is not essential, either. Thorpe[4] showed that Ball's method applies equally well to a continuously stratified fluid, and Thorpe's discussion may be modified to show that resonant triads involving one surface and two internal waves exist in a continuously stratified fluid. Finally, these triads persist in the Boussinesq limit [obtained by setting $\Delta = 0$ in the coefficient of ω^4 in (2)], because the construction in Fig. 2 exhibits a topological property of the equations. In this limit, (4) becomes

$$(\omega^2 - g|k|)(\omega^2 - \tfrac{1}{2}\Delta g|k|) = 0, \qquad (6)$$

and (5) is replaced by

$$\begin{aligned} \omega_1 &= \left(\frac{2+\Delta}{2-\Delta}\right)\Omega, & g k_1 &= \frac{2}{\Delta}\left(\frac{2+\Delta}{2-\Delta}\right)^2 \Omega^2, \\ \omega_2 &= \Omega, & g k_2 &= -\frac{2}{\Delta}\,\Omega^2, \qquad (7) \\ \omega_3 &= \frac{4}{2-\Delta}\,\Omega, & g k_3 &= \left(\frac{4}{2-\Delta}\right)^2 \Omega^2. \end{aligned}$$

Thus, these triads should be considered common in a density-stratified fluid with a free surface.

The physical consequences of this triad are not yet clear. The system in question is conservative, and the frequencies in (1) are all positive. It follows from a theorem of Hasselmann[5] that in a two fluid system, a uniform train of surface waves of small amplitude (a) is unstable; any tiny perturbation of the internal waves will grow (exponentially, at first) at the expense of the surface wave. The initial growth rate, λ, of the disturbance is proportional to $(\omega_1 \omega_2)^{\frac{1}{2}} |D||a|$, where D is the interaction coefficient (which is not calculated here). Because $\omega_1^2, \omega_2^2 \sim O(g\Delta)$ from (3), it follows that the growth rate is on the order of

$$\lambda = O(\Delta^{\frac{1}{2}} |D||a|) . \tag{8}$$

This growth rate may be compared to that of the Benjamin-Feir[6] instability of short surface waves:

$$\Lambda \sim O(|a|^2) . \tag{9}$$

For configurations in which $D = O(1)$ and $|a| \ll \Delta^{\frac{1}{2}}$, the instability of the surface waves to two internal waves dominates over the Benjamin-Feir instability. Moreover, because the surface wave in this triad tends to be much longer than the two internal waves, this instability is viable even for long surface waves, where the Benjamin-Feir instability is inoperative.

On the other hand, one may show by the same reasoning that one of the two surface waves always is unstable in any of the other triads possible in the two-layer system. The growth rates of these instabilities

also are proportional to the amplitude of the unstable surface wave, as in (8). It is not clear without a detailed calculation of the interaction coefficients which instability dominates. However, the purpose of this note is not to assert the dynamic significance of this triad, but merely its existence.

Acknowledgement

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References

[1] F. K. Ball, J. Fluid Mech. 19, 465 (1964).
[2] O. M. Phillips, Ann. Rev. Fluid Mech. 6, 93 (1974).
[3] H. Lamb, Hydrodynamics § 231, Dover, N.Y. (1932).
[4] S. A. Thorpe, J. Fluid Mech. 24, 737 (1966).
[5] K. Hasselman, J. Fluid Mech. 30, 737 (1967).
[6] T. B. Benjamin and J. E. Feir, J. Fluid Mech. 27, 513 (1967).

Figure Captions

Figure 1: Two layer system, with notation for Eq. (2); ρ denotes the density of the fluid.

Figure 2: Dispersion curves for Eq. (3), with $\Delta = 0.02$, and the geometric construction of a resonant triad involving one surface (B) and two internal (A, C) waves.

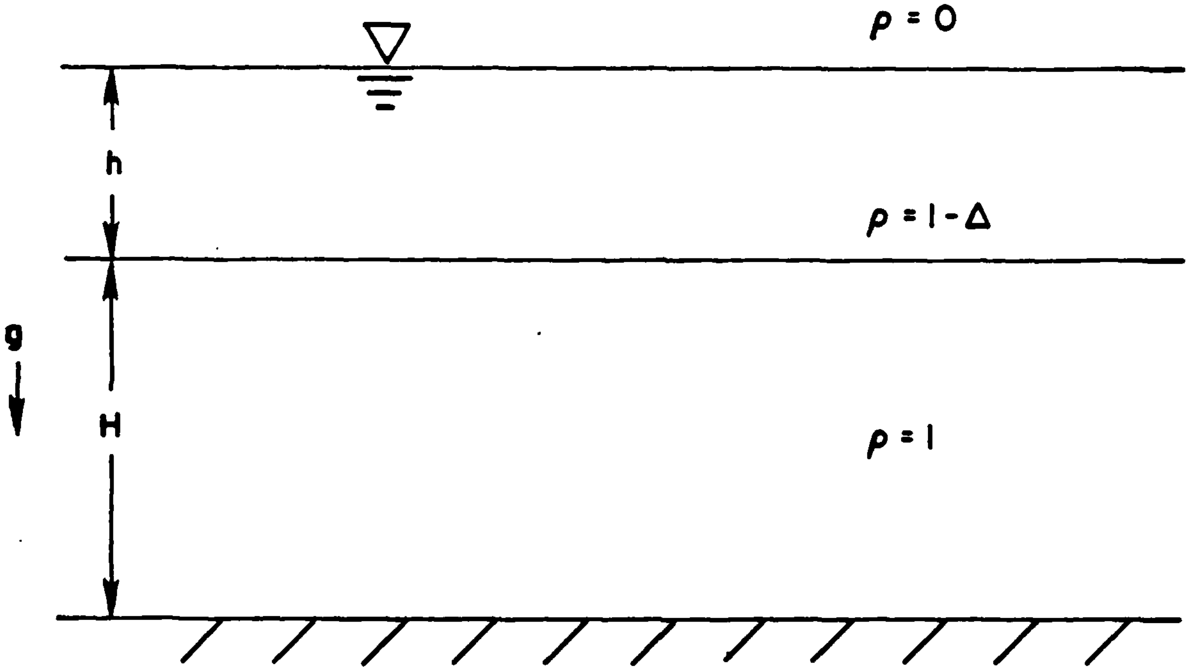

Figure 1

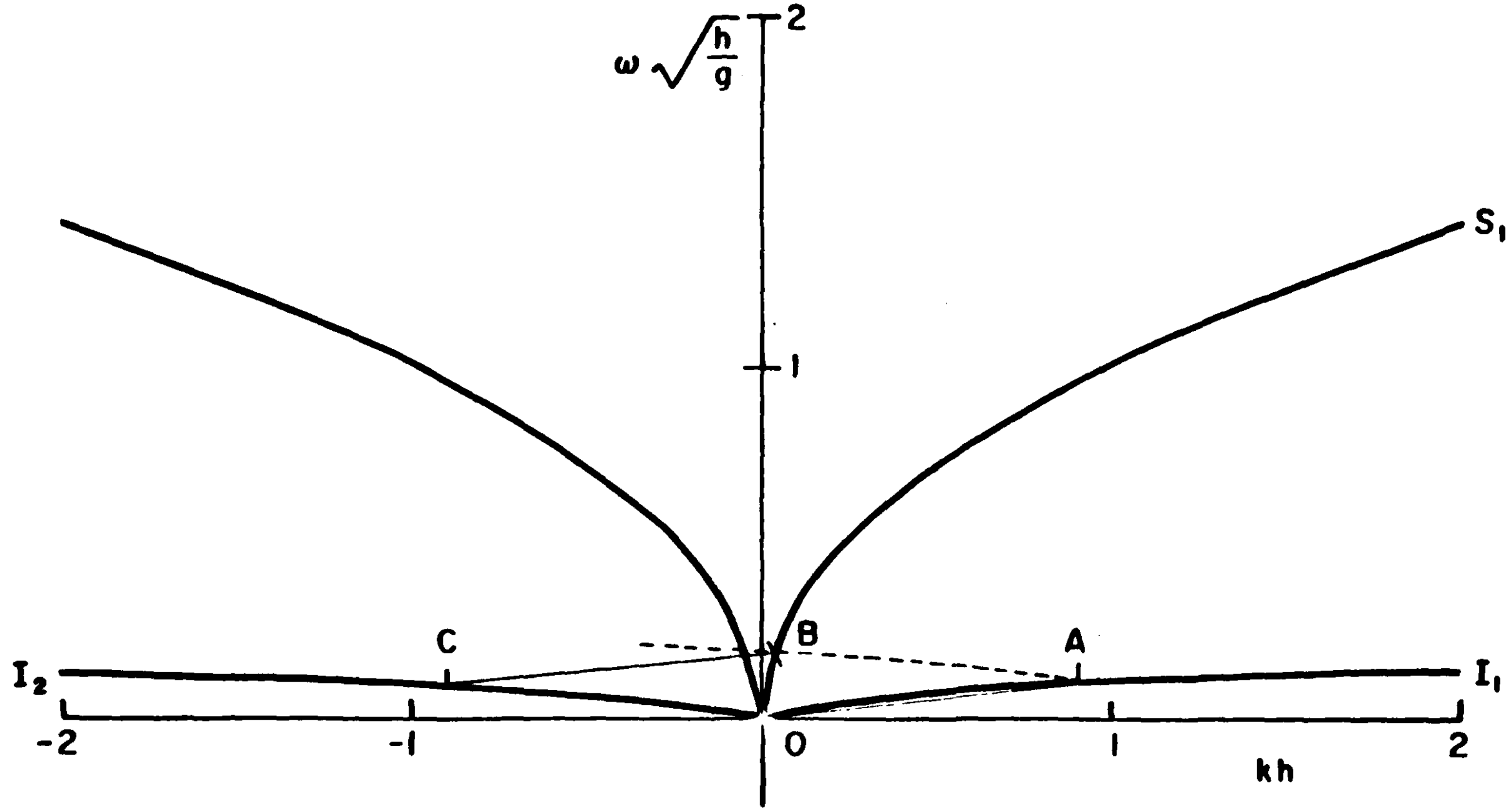

Figure 2